# 1-3/1-5 Exponential and Logarithmic Functions

$y = a \cdot b^x$ (a: amount after a certain time; initial amount; b: rate; x: time; $b > 1$ growth; $0 < b < 1$ decay)

**Learning Objectives:**

- I can use the rules of exponents to simplify an expression.
- I can use the rules of logarithms to simplify an expression.
- I can model exponential growth and decay using a function.
- I can evaluate logarithms.
- I can solve an exponential equation.
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## Exponential Growth and Decay Formulas

| Growth/Decay by Rate | Natural Growth |
|---|---|
| $y = A(1 \pm r)^t$ | $y = Ae^{kt}$ |
| A = Initial amount | A = Initial amount |
| r = growth/decay rate | k = growth/decay constant ($k>0$, $k<0$) |
| t = time | t = time |
| y = Amount after time t | y = Amount after time t |

$A = Pe^{rt}$

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$e \approx 2.71828182843.....$

*e* is an irrational number (like π)

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## Compound Interest

$$y = A\left(1 + \frac{r}{n}\right)^{nt}$$

A = Initial amount

r = growth/decay rate

n = number of times per year that interest is compounded

t = time (in years)

y = Amount after time t

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**Ex1.** A 100 gram sample of a radioactive substance decays so that there is a 75 grams left after 10 days

$y = 100e^{kt}$

$75 = 100e^{k(10)}$

$75 = 100e^{10k}$

a.) Find the decay constant and write an exponential decay model of the form $y = Ae^{kt}$

$\ln .75 = \ln e^{10k}$

$\ln .75 = 10k$

$k = -.029$

$y = 100e^{-.029t}$

b.) Find the amount left after 50 days

23.731 g

c.) Find how many days it will take for there to be 10 grams of the substance left?

$10 = 100e^{-.028768t}$

$.1 = e^{-.028768t}$

$\ln .1 = \ln e^{-.028768t}$

$\ln .1 = -.028768t$

$t = 80.0398$ days

d.) Find the length of time for one half-life.

$50 = 100e^{-.028768t}$

$t = 24.094$ days

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## Laws of Exponents

1. Product Property $b^m \cdot b^n = b^{m+n}$ $x^2 \cdot x^5 = x^7$
2. Quotient Property $\frac{b^m}{b^n} = b^{m-n}$ $\frac{x^5}{x^2} = x^{5-2} = x^3$ $\frac{x^2}{x^5} = \frac{1}{x^3}$
3. Power Property $(ab)^m = a^m b^m$ $(3x^2)^5 = 3^5 x^{10} = 243x^{10}$
4. $(b^m)^n = b^{mn}$ $x^{14} = (x^2)^7$
5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ $\left(\frac{x}{2}\right)^4 = \frac{x^4}{16}$

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p. 26 #5) $9^{2x}$, $(3^2)^{2x}$, $3^{4x}$

p. 26 #23) $y = a \cdot b^t$

$y = 6.6\left(\frac{1}{2}\right)^{t/14}$

6. $1 = 6.6\left(\frac{1}{2}\right)^{t/14}$

$\frac{1}{6.6} = \left(\frac{1}{2}\right)^{t/14}$

$y = 6.6e^{k \cdot t/14}$

$\ln\left(\frac{1}{6.6}\right) = \frac{t}{14} \ln .5$

#31) $y = ab^x$

$y = 1 \cdot 2^{48}$

#44) $y = 2e^{-x} - 3$

$y = \frac{2}{e^x} - 3$

p. 19 #57) $f(x) = x^4 + x^2 + x$

$f(-x) = f(x)$

$f(-x) = (-x)^4 + (-x)^2 + (-x)$

$f(-x) = -f(x)$

Sep 14-1:24 PM

# Logarithms

$E = \log_b N \quad \rightleftarrows \quad b^E = N$

$3 = \log_2 8 \quad \rightleftarrows \quad 2^3 = 8$

A Log is an Exponent!

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Ex2. Evaluate Each logarithm without the use of a calculator

1.) $\log_5 25$ $5^{\square}=25$ (2)

2.) $\log_3 \frac{1}{9}$ -2

3.) $\log_{36} 6$ $\frac{1}{2}$

4.) $\log_{27} 3$ $\frac{1}{3}$

5.) $\log_4 8$ $4^{\square}=8$ $\frac{3}{2}$

6.) $\log_{16} \frac{1}{8}$ $-\frac{3}{4}$

7.) $\log_{13} 1$ 0

8.) $\log_{10} 0$ und.

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# Laws of Logs

- Product Property

$$\log(AB) = \log A + \log B$$

$\log(3x) = \log 3 + \log x$

- Quotient Property

$$\log\left(\frac{A}{b}\right) = \log A - \log B$$

$\log\left(\frac{3}{x}\right) = \log 3 - \log x$

- Power Property

$$\log M^k = k \log M$$

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Simplify the following expressions:

a) 7logx-2logx $\log\left(\frac{x^7}{x^2}\right) = \log x^5$ $5\log x$

b) 2logx+3logx $5\log x$ $\log(x^2 \cdot x^3) = \log x^5$

c) (logx-1)(logy+3) $\log x \log y + 3\log x - \log y - 3$

d) 4logx+8logy-12logz $\log\left(\frac{x^4 y^8}{z^{12}}\right)$

Aug 28-10:08 AM

# Common vs Natural Logs

**Common Log (base 10)**

$\log_{10} x = \log x$

**Natural Log (base e)**

$\log_e x = \ln x$


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Ex3. Solve each equation

1. $3^x = 7$ $\log 3^x = \log 7$ $x = \frac{\log 7}{\log 3} = 1.771$

2. $e^x = 271$

3. $4e^x + 7 = 178$ $4e^x = 171$ $e^x = \frac{171}{4}$ $x = 3.755$

4. $\log_8 \sqrt{1-x} = \frac{1}{3}$ $8^{1/3} = \sqrt{1-x}$ $1-x=4$ $x=-3$

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## Change of Base Formula

$\log_b M = x$

$b^x = M$

$\log(b^x) = \log(M)$

$x \log b = \log M$

$x = \frac{\log M}{\log b}$

$\log_2 5 = \frac{\log 5}{\log 2}$

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Ex4. Evaluate

$\log_6 27 = 1.839$

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## Homework

pg 26 # 1, 2, 5, 9, 19, 21, 23, 24, 26, 31, 43 – 45

pg 44 # 13, 15, 16, 18, 22
# 33, 34, 37, 41, 46, 48, 54-57

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